Ant in the Well

An ant is at the bottom of a 12-foot deep well and is trying to get to the top. During the day he climbs 4 feet up, but at night he slides back 2 feet. How long does it take for him to get out of the well?

Explain your solution with words and a drawing.

Ant in the Well

Suggested Grade Span

3-5

Task

An ant is at the bottom of a 12-foot deep well and is trying to get to the top. During the day he climbs 4 feet up, but at night he slides back 2 feet. How long does it take for him to get out of the well?

Explain your solution with words and a drawing.

Alternate Versions of Task

More Accessible Version:

An ant is at the bottom of an 8-foot deep well and is trying to get to the top. During the day he climbs 4 feet up but at night he slides back 2 feet. How long does it take for him to get out of the well?

Explain your solution with words and a drawing.

More Challenging Version:

An ant is at the bottom of a 5-foot deep well and is trying to get to the top. During the day he climbs up 4 1/2 inches, but at night he slides back down 2 3/4 inches. How long does it take for him to get out of the well?

Explain your solution with words and a drawing.

Context

Students may approach this problem a variety of ways. Some will bring to it a basic understanding of division. Others will use a diagram to solve the problem. Some will solve it using addition or subtraction, although the use of a diagram allows them to see that the ant gets to the top of the well on the fifth day, and does not need to fall back.

What This Task Accomplishes

The strength of the problem is that a solution with a diagram will clearly bring out its underlying assumption (the ant does not have to fall back on the fifth day).

This problem allows students to see that an algorithmic solution (2 + 2 + 2 + 2 + 2 + 2 = 12 or 12/2 = 6) is really not complete. Many students will want to solve this problem without drawing a mathematical representation. The task can lead to a good discussion of the importance of clearly understanding the assumptions lying behind problems by drawing a diagram.

What the Student Will Do

Some students will jump right in with an algorithm, others will attempt a diagram. Solutions are best that combine the two, along with an explanation of their reasoning.

Time Required for Task

60 minutes

Interdisciplinary Links

There are none. This problem focuses on mathematics and representations.

Teaching Tips

If a student comes immediately to an algorithmic solution, ask the student to take a different approach, using a mathematical representation. Teachers may want to take time to allow students to discuss their different approaches to solving this problem.

Suggested Materials

Graph paper

Possible Solutions

Although some students will conclude that the ant can get out of the well in six days, that answer, usually arrived at algorithmically, is not the most appropriate solution. There may be a few students with a good explanation, for example, that the ant was too tired to complete his journey in five days, so he fell back and climbed out at the beginning of the sixth day. By using a mathematical representation, students will conclude that the ant can complete his journey in five days.

More Accessible Version Solution:

Day	Day Progress (+4)	Night (-2)	Where Landed
1	4	-2	2
2	6	-2	4
3	10	-2	8

The ant would technically get out of the well at the end of day three before nighttime begins.

More Challenging Version Solution:

5 feet x 12 inches = 60 inches to get out of the well.

Day	Day Progress (+4.5")	Night (-2.75")	Where Landed (inches)
1	4.5	-2.75	1.75
2	6.25	-2.75	3.5
3	8	-2.75	5.25
4	9.75	-2.75	7
5	11.5	-2.75	8.75
6	13.25	-2.75	10.5
7	15	-2.75	12.25
8	16.75	-2.75	14
9	18.5	-2.75	15.75
10	20.25	-2.75	17.5
11	22	-2.75	19.25
12	23.75	-2.75	21
13	25.5	-2.75	22.75
14	27.25	-2.75	24.5
15	29	-2.75	26.25
16	30.75	-2.75	28
17	32.5	-2.75	29.75
18	34.25	-2.75	31.5
19	36	-2.75	33.25
20	37.75	-2.75	35
21	39.5	-2.75	36.75
22	41.25	-2.75	38.5
23	43	-2.75	40.25
24	44.75	-2.75	42
25	46.5	-2.75	43.75
26	48.25	-2.75	45.5
27	50	-2.75	47.25
28	51.75	-2.75	49
29	53.5	-2.75	50.75
30	55.25	-2.75	52.5
31	57	-2.75	54.25
32	58.75	-2.75	56
33	60.5	-2.75	57.75

On day 33, the ant crawls out of the well before nighttime.

Task Specific Assessment Notes

Novice

There are two examples of Novice solutions here. In each case the students applied inappropriate concepts and procedures and used strategies that did not help solve the problem. One student multiplied (12 x 2), instead of dividing. The second Novice used a diagram, but with the wrong procedure. There is no evidence of mathematical reasoning, they do not tell us why they took the approaches they did. One reason for this is the students do not communicate their thinking. One student used inappropriate mathematical representation.

Apprentice

The solution is not complete. The student uses a strategy that is partially useful leading some way toward a solution, but not to a full solution of the problem. There is some evidence of mathematical reasoning showing us his/her thinking (4 feet - 2 feet = 2 feet). There is some use of mathematical representation, but it is not clearly presented (12|34|56|78|91|01|12). S/he divides 12 numbers into seven equal parts. This conflicts with his/her conclusion that the ant can climb out of the well in six days.

Practitioner

This student demonstrates a broad understanding of the problem and the major concept that the ant can get out of the well in five days. The student's strategy (drawing a diagram) leads to a solution. The diagram shows his/her effective mathematical reasoning. The clear explanation is found in the appropriate use of the accurate mathematical representation.

Expert

An Expert on this problem will make other observations about the problem beyond the solution. Not only does this student show an understanding of the problem, but makes an interesting elaboration by converting two feet each day to one inch of progress per hour. Mathematical procedures are applied accurately and the student uses an efficient strategy to solve the problem. The mathematical representation is actively used as a means of communicating ideas and the original color-coded mathematical representation showed all of the steps so the reader does not need to infer how and why the student made decisions as s/he moved to a solution.